

BUSINESS CYCLE ASYMMETRIES IN STOCK RETURNS: ROBUST EVIDENCE

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Abstract: In this study we employ augmented and switching time series models to find possible existence of business cycle asymmetries in U.S. stock returns. Our approach is fully parametric and testing strategy is robust to any conditional heteroskedasticity, and outliers that may be present. We also approximate in sample as well as out-of-sample forecasts from artificial neural networks for testing business cycle nonlinearities in U.S. stock returns.

Our results based on nonlinear augmented and switching time series models show a strong evidence of business cycle asymmetries in conditional mean dynamics of U.S. stock returns. These results also show that conditional heteroskedasticity is unimportant when testing for asymmetries in conditional mean. Moreover, the conditional volatility in stock returns is asymmetric and is more pronounced in recessions than in expansion phase of business cycles. Similarly, the results based on neural network models show a statistically significant evidence of business cycle nonlinearities in US stock returns. The magnitude of these nonlinearities is more obvious in post World War II era than in the full sample period.

Keywords: asymmetries; business cycles; conditional heteroskedasticity; long memory; nonlinearities; outliers; excess returns; stable distributions

JEL Classifications: C22, C32, G19;

1.Introduction

The possible existence of non-normality in empirical distribution of stock returns and persistence in stock returns volatility has been tested extensively in the literature. French et al. (1987), Akgiray and Booth (1988), Jansan and de Varies (1991), Buckel (1995), and McCulloch (1997) concluded that non-normality do exist in stock returns. Nelson (1991), Danielsson (1994), Pagen and Schwartz (1990), Diebold and Lopez (1995), and Ghose and Kroner (1995)

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demonstrate that volatility persists in the stock returns especially in short run. Similarly, Schwert (1989), Hamilton and Lin (1996), and McQueen and Thorely (1993) showed that conditional volatility in stock returns exists which is counter-cyclical, and this behavior is more pronounced in recession than in expansion phase of business cycles.

Detecting nonlinearities in business cycles is important for several reasons. Nonlinearities imply that the effects of expansionary and contractionary monetary policy shocks on output and other economic and financial variables are not symmetric. Therefore, nonlinearities would invalidate measures of the persistence of monetary policy and other shocks on output and other economic and financial variables that are based on linear models. Behavior of other economic and financial variables including stock returns during business cycles in addition to periods of high and low stock volatility is important. It might be useful to identify financial variables including stock returns that can be candidate variable to predict future recessions.

Although a number of models² were introduced and employed in business cycle research, there appears an ample need for additional non-linear time series models in this area. Therefore, in addition to using nonlinear and switching time series models, we use artificial neural networks (ANN) that would be advantageous in the present study because ANN are used in variety of problems due to their pattern recognition ability, intrinsic capability to use arbitrary input output mapping, and potential to approximate any continuous function with desired level of precision (Hornik et al. 1989). Neural networks have been applied successfully in engineering, medical science, business and economics³ but only a few studies focused on

² Auerbach (1982), Gordon (1986), Kling (1987), Koch and Rasche (1988), Diebold and Rudebusch (1989), Hamilton (1989), Klein (1990), Estralla and Mishkin (1998) including others.

³ Kuan and White (1994) discussed neural networks and their applications in economics. Swanson and White (1995, 1997 a, 1997b) found usefulness of neural network models in economic time series data pertaining to interest rate, unemployment and GNP etc. Similarly, Hutchinson et al. (1994) employed neural networks in option pricing, Garcia and Gencay (2000),

business cycles⁴. We believe that the study of business cycles in the area of stock returns might benefit from additional nonlinear models, especially neural networks. We therefore, employ artificial neural network tests to find possible existence of nonlinearities in U.S. stock returns in addition to various time series models.

During the past few decades, a numbers of studies have been undertaken in context of the business cycles⁵; however, these studies focus only on macroeconomics time series data⁶ although there appears ample need to find business cycle asymmetries in other variables of prime importance including stock returns. We feel that the present study can fill this gap adequately and the study of business cycles asymmetries in stock returns will benefit this area of research. Moreover, in the present study we employ a number of nonlinear and switching time series models that fully account for conditional heteroskedasticity⁷, and outlier⁸ in the data series in addition to artificial neural networks (ANN) that might help investigating possible existence of business cycle asymmetries in stock returns adequately.

Gencay (1998), and Qi and Madala (1999) in stock market predictions, and Gencay (1999) in exchange rate forecasting.

⁴ Vishwakarma (1994), Qi (2001), Kiani (2005), and Kiani, Bidarkota, and Kastens (2005)

⁵ Neftci (1984), Brunner (1992, 1997), Beaudry and Koop (1993), Potter (1995), and Ramsey and Rothman (1996), Anderson and Vahid (1998), Bidarkota (1999-2000), Anderson and Ramsey (2002), Kiani and Bidarkota (2004), Kiani (2005), and Kiani, Bidarkota, and Kastens (2005) including many others.

⁶ Study by Perez-Quiros and Timmermann (2001) is an exception

⁷ French and Sichel (1993) and Brunner (1992, 1997) show existence of conditional heteroskedasticity in real GNP data, and Granger (1995) recommends that linearity be tested using heteroskedasticity-robust tests.

⁸ Tsay (1988) demonstrate that nonlinearities reported in various studies are due to presence of outlier in the data. Balke and Fomby (1994) and Scheinkman and LeBaron (1989) showed that presence of outlier weakens the evidence of nonlinearities. Bidarkota (1999, 2000), Perez-Quiros and Timmermann (2001), and Kiani and Bidarkota (2004) report strong evidence of nonlinearities even after accounting for outlier in the data.

In this study we investigate possible existence of business cycle asymmetries in conditional mean dynamics of U.S. stock returns and real GDP growth rates using a number of nonlinear and regime switching time series models that encompass conditional heteroskedasticity to account for time varying volatility and stable distributions to account for outliers in the series that may be present. We also use tests constructed from in-sample as well as jackknife out-of-sample approximations from artificial neural networks to find possible existence of nonlinearities in U.S. stock returns.

The remaining study is organized as follows. Section 2 discusses artificial neural networks, and various time series models employed in this study. Section 3 provides details on the data, empirical results, specification search, parameter estimates and neural network tests results. Statistical tests for asymmetries and other hypotheses of interest are reported in detail in Section 4. Important conclusions that can be drawn from this study are reported in Section 5.

2. Empirical Models

In this study we employ nonlinear time series models and artificial neural networks to find possible existence of business cycle asymmetries in the series. We can choose our model parameterization using Schwarz Bayesian Criterion (SBC) due to Schwarz (1978) because Akaike Information Criterion (AIC) due to Judge et al. (1985) AIC has a tendency to pick larger models.

2.1. Nonlinear time series model

In this study we employ three classes of nonlinear time series models to find possible existence of business cycle asymmetries in U.S. stock returns. Each class of these models is further sub-divided into three models. Model 1 is the most general model in each class. We obtain model 2 when we impose homoskedasticity in model 1. Similarly, we get model 3, when we restrict errors to come from stable family. In the following sub-sections, we elaborate the most general models in each class.

2.1.1. CDR augmented models. This model was originally proposed by Beaudry and Koop (1993) and incorporates an ad hoc nonlinear term in an autoregressive moving average (ARMA) framework. Bidarkota (2000) and Kiani and Bidarkota (2004) used a

modified version of this model due to Bidarkota (1999) that incorporates stable distributions, conditional heteroskedasticity, and long memory. However, we use a restricted version of this model not incorporating long memory, which is shown in the following Equations:

$$\Phi(L)(1-L)(\Delta y_t - \mu) = [\Omega(L) - 1]CDR_t + \varepsilon_t \quad (2.1a)$$

$$\varepsilon_t | I_{t-1} \sim z_t c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 |\varepsilon_{t-1}|^\alpha \quad (2.1b)$$

Here, $\Delta y_t \equiv 100 * \Delta(\ln y_t)$ is the growth rate of stock return, its unconditional mean is μ , $\Omega(\cdot)$ and $\Phi(\cdot)$ are polynomials of orders r and p respectively in the lag operator L , with $\Omega(0) = \Phi(0) = 1$. The term CDR_t is defined as $CDR_t = \max\{y_{t-j}\}_{j \geq 0} - y_t$ and it represents the current depth of recession.

We use stable distribution to capture outlier in the data series. A random variable X will have a symmetric stable distribution $S_\alpha(\delta, c)$ if its log characteristic function can be expressed as $\ln E \exp(iXt) = i\delta t - |ct|^\alpha$. Where, $\delta \in [-\infty, \infty]$ is the location parameter, $c \in [0, \infty]$ is the scale parameter, and $\alpha \in [0, 2]$ is the characteristic exponent. When $\alpha = 2$, we obtain the normal distribution. Smaller values of this exponent indicate thicker tails. In Equation 2.1b setting α equal to 2 gives us a normal GARCH (1,1) process in conditional volatility.

Our model in Equation 2.1b is similar to that power ARCH model in Ding, Granger and Engle (1993). However, there is a difference in the later specification that the distribution of z_t does not depend on the characteristic exponent α . Similarly McCulloch (1985) fitted a GARCH-Stable model to bond returns using absolute values instead of α power. However, Liu and Brorsen (1995) modeled volatility of the daily foreign returns using stable errors.

Although we have an ad hoc non-linear CDR_t term within an otherwise standard AR framework, this model is simple and parsimonious. When, $\Omega(L) = 1$, Equation 1a reduces to an

autoregressive (AR) model. Since it nests AR models, we can use the standard t -statistic or the likelihood ratio (LR) statistic to test the statistical significance of the non-linear term governing the conditional mean dynamics. However, the asymptotic distribution of the t -test for the significance of the non-linear CDR_t term in the model given by Equation 2.1 is non standard (Hess and Iwata, 1997), both when the dependent variable is non-stationary [i.e. integrated of order one $I(1)$], and when it is stationary [$I(0)$]. With $\Omega(L) \equiv 1 + \omega_1 L + \omega_2 L^2 + \dots + \omega_r L^r$, when the autoregressive lag order p is 0 and r is 1, $\omega_1 = 0$ yields a random walk with drift. However, a positive ω_1 implies that negative shocks are less persistent whereas a negative ω_1 implies that positive shocks are less persistent.

Asymmetries means that either the innovations are asymmetric but the impulse transmission mechanism is linear, or that the innovations are symmetric but the impulse transmission mechanism is nonlinear, or both. However, it would be hard to disentangle the asymmetric innovations from the nonlinear propagation mechanism, if they both exist in a data series.

Although asymmetric α -stable distributions exist and are well defined, to determine whether asymmetries in the conditional mean dynamics of the real stock returns growth rates are caused by asymmetric impulses being propagated linearly or symmetric impulses being propagated nonlinearly or asymmetric impulses being propagated nonlinearly is beyond the scope of this study. Here, we are merely investigating whether asymmetries exist in the conditional mean regardless of how they can best be characterized.

2.1.2. CDR -switching models. We employ modified version of the switching models proposed by Beaudry and Koop (1993) that were also employed by Kiani and Bidarkota (2004). These modifications due to Bidarkota (1999), include, stable distributions, conditional heteroskedasticity, and long memory. However, we restrict long memory in these modes. The most general model in this class of models without long memory component is shown in the following Equations:

$$\text{In Regime 1: } (1 - \phi_1 L - \phi_2 L^2)(1 - L)(\Delta y_t - \mu_1) = \varepsilon_t \quad (2.2a)$$

$$\varepsilon_t | I_{t-1} \sim z_t c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 | \varepsilon_{t-1} |^\alpha \quad (2.2b)$$

In Regime 2:

$$(1 - \phi_3 L - \phi_4 L^2)(1 - L)(\Delta y_t - \mu_2) = \varepsilon_t \quad (2.2c)$$

$$\varepsilon_t | I_{t-1} \sim z_t \gamma c_t, \quad z_t \sim \text{i.i.d. } S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 | \varepsilon_{t-1} / \gamma |^\alpha \quad (2.2d)$$

When $CDR_{t-1} = 0$, we get regime 1 and when $CDR_{t-1} > 0$ we obtain regime 2. The unconditional mean of the process and the AR coefficients in the two regimes are different. The parameter γ in regime 2 shows that the model has different scales in the two regimes as well.

2.1.3. SETAR-Switching Models. The SETAR-Switching model was originally proposed by Potter (1995). Bidarkota (2000), and Kiani and Bidarkota (2004) used a modified version of this model due to Bidarkota (1999) that includes, stable distributions, conditional heteroskedasticity and long memory. The most general model estimated within this class of models without long memory is shown in the following Equations.

In Regime 1:

$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)(\Delta y_t - \mu_1) = \varepsilon_t \quad (2.3a)$$

$$\varepsilon_t | I_{t-1} \sim z_t c_t \quad z \sim \text{iid} \quad S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 | \varepsilon_{t-1} |^\alpha \quad (2.3b)$$

In Regime 2:

$$(1 - \phi_3 L - \phi_4 L^2)(1 - L)(\Delta y_t - \mu_2) = \varepsilon_t \quad (2.4c)$$

$$\varepsilon_t | I_{t-1} \sim z_t \gamma c_t \quad z \sim \text{iid} \quad S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 | \varepsilon_{t-1} / \gamma |^\alpha \quad (2.5d)$$

When $\Delta y_{t-2} > 0$, we get regime 1 and when $\Delta y_{t-2} \leq 0$, we get regime 2.

2.2. Artificial neural network

Artificial neural network (ANN) consists of an advanced artificial intelligence technology that mimics human brain's learning and decision-making process. ANN consists of a number of interconnected elements called neurons or nodes. The ability of information processing makes ANN capable to be powerful computational devices that can learn from examples and generalize learning to solve problems never seen before (Rilley and Cooper 1990). ANN modeling approach is useful for forecasters and researchers especially in problems where data is available but the data generating process as well as the underlying laws for data generating process are unknown. ANNs are treated as nonlinear, nonparametric statistical methods due to which these are independent of distributions of the underlying data generating processes (White 1989).

Since 1980s the researchers have developed numerous dissimilar ANN models. One of the most influential neural networks models is the multilayer perceptrons (MLP), which consists of several layers of nodes. This type of neural networks is used for a variety of problem especially forecasting because of their intrinsic capability to use arbitrary input output mapping. The input node is the lowest node where information from sources external to the neural networks is included. The output layer is the highest layer where the solution of the problem is realized. Input and output layers are separated by a number of layers called hidden nodes (Zhang et al. (1998). ANN are able to approximate any continuous function with a desired level of precision (Hornick et al. 1989). A typical single layer feed forward neural networks as of Lee, et al. (1993) can be written as follows.

$$f(x, \mathbf{x}) = \mathbf{q}_0 + \sum_{i=1}^k \mathbf{q}_i \{ \mathbf{y}(\mathbf{g}_i' \mathbf{w}_i) \}, \quad k \in N \quad (2.1)$$

Where, $\mathbf{w}_t = (1, \tilde{\mathbf{w}}_t')'$, and $\tilde{\mathbf{w}}_t = (y_{t-1}, \dots, y_{t-p})'$. Equation 2.1 shows flexible functional forms (Lee et al. (1993), and White (1989)) where \mathbf{y} is a transfer function. The transfer function can be either sigmoid (logistic) or hyperbolic (tangent) cumulative distribution function.

Although neural network models are under criticism for over-fitting (Norwood et al. 2003), neural networks literature contains a number of rules of thumb to select the number of nodes in a neural networks model which solves over-fitting problem. For example Lippman (1987) suggests $2n + 1$ nodes, Wong (1991) proposes $2n$, Tang and Fishwick (1993) suggest n and Kang (1991) recommends ' $n/2$ ' nodes to be added in a neural networks when n is the number of input nodes. However, it is hard to say if any or all these rules of thumb would be adequate for all types of real life problems. According to Zhang et al. (1998) the best way to select the number of nodes in a neural networks model is trial and error, however, Kastens et al. (1995), and Kiani (2005) recommend hidden nodes to be as low as two to avoid over-fitting.

A single hidden layer neural networks model is sufficient to approximate any complex nonlinear function with any desired accuracy (Zhang et al. 1998). Most authors use one single hidden layer for forecasting purposes but it may require a large number of nodes, which is not desirable because it would worsen the network generalization and increase in training time unnecessarily. Neural networks with fewer hidden nodes are desirable for neural network tests because the test will have less power to reject linearity when more hidden nodes will be used. Therefore, our neural network models include as low as two nodes.

2.3. Jackknife re-sampling

Researchers often use jackknife re-sampling technique⁹ when the distribution of the parameters under review is either unknown, when it cannot be characterized by a mathematical function, or when the mathematical function is especially difficult to estimate. The standard jackknife estimate is calculated deleting one observation and estimating parameters using $n - 1$ observations of the data. The model and the observations are then used to conditionally predict the dependent values for deleted observation. The process continues detecting and predicting different observations for each model, until each of the observation in the data had been predicted out-of-sample.

⁹. Quenouille (1949), Tuckey (1958).

Compared to the standard jackknife, in the sub-sample jackknife¹⁰, more than one observation is dropped to estimate out-of-sample forecast of the remaining $m = n - d$ observations, where, n is the total number of observations and $d = 2, 3, \dots, n-1$. The model and the estimated observations are then used to conditionally predict value for the deleted observation and this process continues until each observation in the data is predicted or until all possible sub-samples are considered, depending on the statistical assumptions and the application.

We approximated out-of-sample forecasts for U.S. stock returns, and real GDP growth rates using sub-sample jackknife re-sampling technique based on linear and artificial neural networks wherein we deleted arbitrarily three observations (to save computer time) to estimate the out-of-sample forecasts of the fourth observation. Repeating this process recursively, we estimated out-of-sample forecasts of all the data points for each of the series studied.

2.4. Neural network tests

The neural network test detects possible existence of nonlinearities in stock returns. This test was proposed by Terasvita et al. (1993) and subsequently employed by Kiani et al (2005) and Kiani (2005) and many others. The test is based on the null hypothesis of linearity against an alternative hypothesis of nonlinearity, and it consists of the following two equations:

$$y_t = \pi' w_t + u_t \quad (2.6)$$

where, $u_t \sim Nid(0, S^2)$, $w_t = (1, \tilde{w}_t)'$, $\tilde{w}_t = (y_{t-1}, \dots, y_{t-p})'$

and $p = (p_0, p_1, \dots, p_p)'$

$$u_t = p' w_t + \sum_{j=1}^k q_{0j} \{y(g_j' w_t)\} + v_t \quad (2.7)$$

Where, $\psi(\gamma' w_t) = (1 + \exp\{-\gamma' w_t\})^{-1}$ and p_0 is intercept. Equation 2.7 shows a nonlinear neural network model that nests linear model represented by Equation 2.6. The test consists of three-step procedure. In the first step we regress (Equation 2.6) U.S. stock

¹⁰ Wu (1990), Politis and Romeo (1994), Politis et al. (1997), and Ziari et al. (1997).

returns/ real GDP growth rates (y_t) on an intercept and lags (y_{t-1}, \dots, y_{t-k}) and recover residual (\hat{u}_t). In the second step we employ residual (\hat{u}_t) in conjunction with lags on U.S. stock returns/ real GDP growth rates (y_{t-1}, \dots, y_{t-k}) for neural network approximations (Equation 2.7) to recover residuals (\hat{v}_t). Finally, we compute test statistic using Equation 2.8.

$$TS = \{(SSE1 - SSE2)/m\} / \{SSE2/(n - p - m - 1)\} \quad (2.8)$$

Where, in Equation 2.8, m denotes the number of restrictions in the unrestricted model, n is the number of observations, and p is the number of lags. The test statistics is distributed approximately F under normality hypothesis with $(n-p-m-1)$ and m degrees of freedom. This test statistics is approximate because of the nuisance parameter that appears under the alternative hypothesis (Davis, R. 1977; Andrews, W. 2001).

2.5. Estimation issues

Beaudry and Koop (1993) simply included an additive nonlinear term in a standard autoregressive moving average (ARMA) model to capture nonlinearities in business cycles assuming that the shocks are normally distributed. Although addition of a nonlinear term as such is ad hoc, it does not pose any estimation problems. Bidarkota (1999, 2000) and Kiani and Bidarkota (2004) successfully used this model without any moving average terms but with errors having a more general stable distribution, conditional heteroskedasticity, and long memory.

We restrict ourselves to symmetric stable distribution because maximum likelihood estimation of mixed ARMA models with stable distributions poses a challenge although maximum dispersion estimator (Brockwell and Davis 1991) and Whittle estimators (Mikosch et al. 1995) have been proposed for such situations. The reason we employ symmetric stable distribution in this study is straightforward. We employ computational algorithms developed by McCulloch (1997) to obtain stable densities for maximum likelihood estimation of our models. This algorithm works only when the errors are symmetric stable.

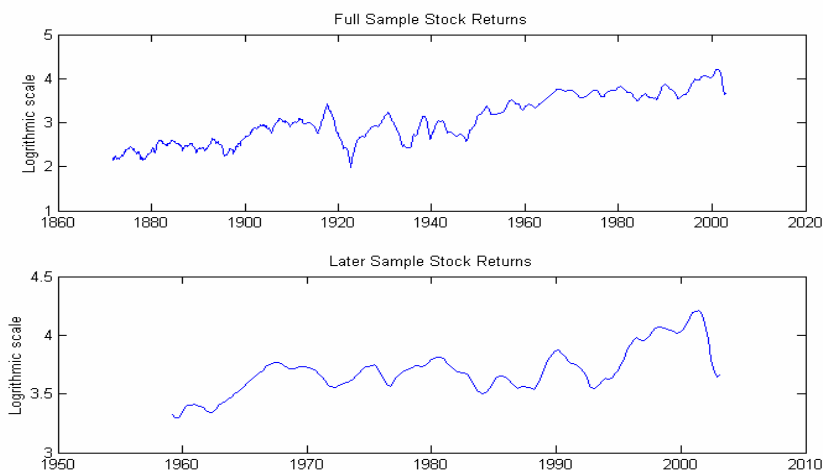
Sowell (1992) proposed full information maximum likelihood (ML) method to estimate ARFIMA models when errors are iid normal. However, for more complicated non-normal conditionally heteroskedastic models, we use conditional sum of square (CSS) estimators. The CSS estimators were originally proposed by Hosking (1984) to estimate ARFIMA-GARCH models with normal and Student-t errors. The CSS procedure is equivalent to the full information MLE asymptotically. Baillie et al. (1996) used conditional sum of square (CSS) estimators and discussed their properties in terms of ARFIMA-GARCH models, particularly with respect to bias. Thus they concluded that CSS method is computationally feasible for complex methods when compared to Sowell's (1992) full information exact MLE.

3. Empirical results

3.1. Data sources

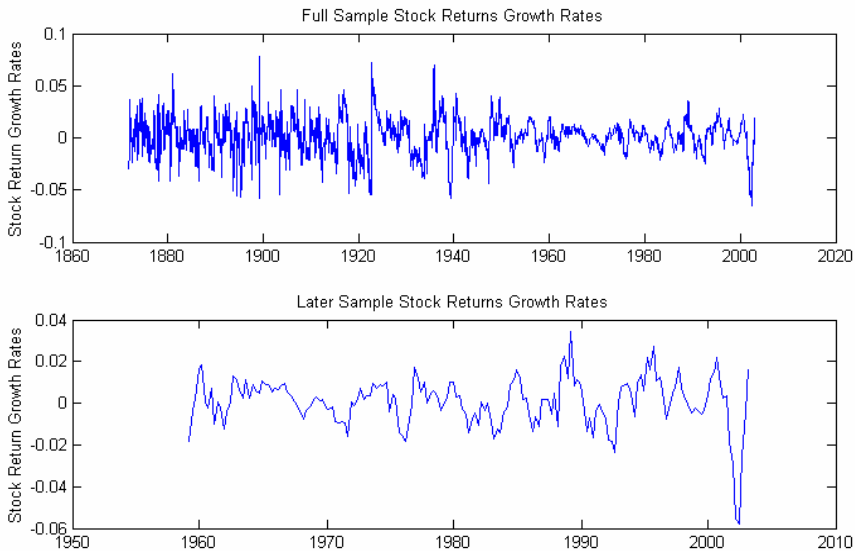
Figure 1 plots logarithmic data series whereas Figure 2 plots growth rates for all the series. We obtained data from Shiller's website¹¹ and calculated quarterly returns from it to use in this study.

Figure 1 Logarithmic Real Excess Return



¹¹ The data is available at <http://www.econ.yale.edu/~shiller/data.htm>.

Figure 2 US Real Stock Returns



The data for full sample period ranges from 1871:1 to 2003:1 and for later sample the date span ranges between 1957:1 to 2003:1 to match with quarterly real GDP series obtained from June 2002 version of International Financial Statistic (IFC)’s CD-ROM. Table 1 in Annex 1 provides additional detail on the data.

3.2. Specification search

An extensive specification search for full sample stock returns was conducted for the three versions (Model 1 through Model 3) of each of the three classes of models described in section 2 above. For the CDR-Augmented class of models, the specification search was done over all parameterizations with lag orders for the autoregressive and CDR_t terms of three or less for parsimony. For the two classes of switching models, namely the CDR-Switching and SETAR-Switching models, the search was done with the autoregressive lag polynomials in the two regimes restricted to be of orders (3,3), (2,2), (1,1), or (0,0). The best parameterizations for each version within each class of models are selected for each series by the minimum Schwarz Bayesian Criterion (SBC). The chosen parameterizations are reported in Table 2. The Table also reports the number of

parameters in the models, the maximized log-likelihood values, and the values of the Akaike information (AIC) and Schwarz Bayesian (SBC) criteria, respectively.

Table 2: Specification Search

	Model 3	Model 2	Model 1
Model parameterization	(3,0)	(2,0)	(2,1)
selected by minimum SBC	(2,2)	(2,2)	(2,2)
Criterion	(2,2)	(2,2)	(3,3)
Number of parameters	5	5	9
	8	9	12
	8	9	14
Log likelihood	-2660.54	-2413.02	-2186.93
	-2655.38	-2402.63	-2184.98
	-2653.59	-2406.33	-2177.23
AIC	5331.07	4836.04	4391.86
	5326.96	4823.27	4393.95
	5323.18	4831.48	4382.47
SBC	5337.88	4862.85	4440.11
	5369.65	4871.52	4458.28
	5366.06	4879.73	4457.52

Notes: 1. For each item listed in the first column, the three rows in the subsequent columns for that item denote the corresponding statistics for the three classes of models CDR-Augmented, CDR-Switching and SETAR-Switching in that order. These models are elaborated in section 2 of the text. 2. The most general model is Model 1. Imposing homoskedasticity on Model 1, we get Model 2. Setting $a=2$ in Model 2 yields Model 3.

3.3. Parameter estimates

Tables 3, contains parameter estimates for the best parameterizations of the most general versions (Model 1) within each of the three classes of models, namely, the CDR-Augmented, CDR-Switching, and SETAR-Switching models. In Table 3 the estimates of the unconditional mean μ , the value of the characteristic exponent α , the volatility persistence parameter b_2 , and ARCH parameter b_3 for CDR-Augmented model, CDR-Switching models, and SETAR-Switching models are shown. The Table also shows estimates of the autoregressive (AR) parameters, and the parameters

and the unconditional mean μ_1 , and m_2 for the switching models (CDR-Switching and SETAR-Switching), and CDR (w) parameter for CDR-Augmented models. For switching class of models, regime 1 is associated with lower mean growth rates for CDR-Switching models and opposite is true for SETAR-Switching models.

Table 3: Parameter Estimates (Most General Model from Each Class)

	CDR-Aug	CDR-Swi	SETAR-Swi
Model Parameterization	(2,1)	(3,3)	(3,3)
μ_1	-0.158 (0.205)	-0.016 (0.070)	0.34 (0.033)
μ_2	-	0.005 (0.017)	-0.002 (0.033)
α	1.598 (0.035)	1.599 (0.039)	1.617(0.081)
c_0	1.404 (0.230)	1.467 (0.249)	1.437 (0.221)
b_1	0.0003 (0.0003)	0.0003 (0.0003)	0.0004 (0.0002)
b_2	0.978 (0.049)	0.970 (0.012)	0.968 (0.006)
b_3	0.0096 (0.007)	0.009 (0.004)	0.008 (0.002)
ϕ_1	0.725 (0.023)	0.632 (0.079)	0.729 (0.035)
f_2	0.172 (0.021)	0.232 (0.074)	0.140 (0.039)
f_3		-0.018 (0.078)	0.747 (0.045)
f_4		0.733 (0.024)	-0.014 (0.027)
f_5		0.161 (0.027)	0.185 (0.418)
f_6		0.011 (0.022)	0.081 (0.032)
w_1	0.001 (0.001)		
γ		1.002 (0.005)	1.016 (0.005)
Log Likelihood	-2186.931	-2184.108	-2177.233

Notes: 1. Parameter estimates are reported for the best model specification for version 1 within the class of CDR-Augmented, CDR-Switching and SETAR-Switching models as determined by the minimum SBC criterion.

The estimates of the characteristic exponent α are very similar for all models. The estimates of the volatility persistence parameter b_2 are uniformly higher and the estimates of the ARCH parameter b_3 are uniformly lower for all the models in the Table. The estimates of unconditional mean μ_1 in regime 1 are close to the values of unconditional mean μ_2 in the regime 2. For both the switching models (CDR-Switching and SETAR-Switching) the estimates of the scale ratio g show that the lower mean regimes are associated with higher volatility, which means that large negative shocks are associated with higher volatility. These results are synonymous to earlier studies including Brannas and De Gooijer (1994).

3.4. Neural network test results

Table 5, in Annex 2, shows results from neural network tests for full sample stock returns as well as sub-sample stock returns and U.S. real GDP growth rates.

4. Hypotheses tests results

Two sets of hypotheses test are employed in this study. The first set is based on time series models, whereas the second pertain to neural network linearity tests. These hypotheses are elaborated in the following sub-sections and presented in Annex 2.

4.1. Hypothesis tests on time series models: 4.1.1 Tests for normality. 4.1.2. Test for Homoskedasticity. 4.1.3. Test for Linearity in Conditional Mean.

4.2. Neural network test

4.3. Empirical results on hypothesis tests: 4.3.1. Results on normality. 4.3.2. Results on homoskedasticity. 4.3.4. Results from neural network tests. 4.3.5. Nature of stock returns.

4.4. Discussion of results on hypotheses tests.

Our results on nonlinearity based on neural network tests as well as on conditional mean for the US stock returns are in line with Quiros and Timmermann (2001), although their testing approach is different to ours. Therefore, the evidence against linearity in mean for the US stock returns is robust to changes in the sample size, sample period and testing approach. Koop and Potter (2001) investigate whether nonlinearities could arise from structural instability. Blanchard and Simon (2001) show a slowdown in the variance of US economic

activity suggesting a possible structural change in the early 1980s. We do not account for this possibility in this study.

5. Conclusions

We employed three classes of augmented and switching time series models, namely, CDR-Augmented, CDR-Switching, and SETAR-Switching models for finding possible existence of business cycle asymmetries in conditional mean dynamics of US stock returns. Our time series models fully account for time varying volatility and outlier that might be present in the series using conditional heteroskedasticity and stable distribution in the data series. Similarly, we use artificial neural networks to approximate in-sample as well as jackknife out-of-sample forecasts for testing nonlinearities in U.S. stock returns and real GDP growth rates.

Results based on nonlinear time series augmented and switching models show a statistically significant evidence of business cycle asymmetries in conditional mean dynamics of U.S. stock returns as well as real GDP growth rates. These results also show that switching models capture nonlinearities better than the augmented models. Similarly, CDR-Switching models show that the stock return volatility in low regimes (recessions) is higher than in the higher regimes (expansions). SETAR-Switching models confirm this behavior which divulges that stock return volatility is higher in recessions than in expansion phase of business cycles.

The results from neural network nonlinearity tests that are constructed from in-sample as well as jackknife out-of-sample forecast approximated from artificial neural network models show robust evidence of nonlinearities in full sample and later sample period stock returns as well as real GDP growth rates. These results also show that the magnitude of nonlinearities in U.S. stock returns in the later sample period is substantially higher than the magnitude of nonlinearities in real GDP growth rates.

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Annex 1

Table 1: Data Description

Data Series	Frequency	Sample Period	Sample Length
US Stock Returns	Monthly	1871:1-2003:1	1577
US Stock Returns	Quarterly	1957:1-2003:1	177
US Real GDP	Quarterly	1957:1-2003:1	177

Notes on Table 1: We obtained real earnings and real stock returns data from Campbell and Schiller's homepage to calculate monthly and quarterly real US stock returns. 2 Data on quarterly real GDP growth rates was obtained from June 2002 version of International Financial Statistic (IFC)'s CD-ROM.

Annex 2.

4. Hypotheses tests results

Two sets of hypotheses test are employed in this study. The first set is based on time series models, whereas the second pertain to neural network linearity tests. These hypotheses are elaborated in the following sub-sections.

4.1. Hypothesis tests on time series models

We performed three types of hypotheses tests on estimated augmented and switching models. These tests include normality test, test for homoskedasticity, and test for linearity in the conditional mean. The following sub-sections describe various tests and sub-section 4.3 provides empirical results on these hypotheses tests.

4.1.1 Tests for normality. The test for normality is based on the value of α . If α equals 2, normality results. If the value of α is less than 2, then the model is non-normal stable. This test compares the likelihood ratio (LR) statistic for two models with identical parameterizations.

Since the null hypothesis for this test lies on the boundary of admissible values for α , the LR test statistic does not have the usual χ^2 distribution asymptotically. Thus the test is based on small sample critical values generated by Monte Carlo simulations reported in McCulloch (1997, Table 4, panel b).

4.1.2. Test for Homoskedasticity. According to the null hypothesis of homoskedasticity the GARCH parameters $b_2 = b_3 = 0$. The test is based on the likelihood ratio test statistic. Since b_1 and c_0 end up being trivial transformations of one another under the null hypothesis, it is not clear whether the LR test statistic asymptotically has 2 or 3 degrees of freedom. We, therefore, base our statistical inference on critical values from the χ^2_3 distribution.

4.1.3. Test for Linearity in Conditional Mean. The test for linearity in the conditional mean is carried out using different versions of the models within only two classes of switching models. This is because the minimum SBC criterion ends up selecting among the four different version of the CDR-Augmented class of models; a version and a parameterization that does not include the additive CDR term in the chosen model except for the most general model (see Table 2). In section 2.5, we reported some added difficulty in testing for the significance of the CDR term with a standard t -test or an LR test.

Table 4: Hypotheses Tests

	Model 3	Model 2	Model 1
LR ($\alpha = 2$)		423.02	232.67
		493.70	553.34
		505.48	239.70
LR (no GARCH)			451.70 (0.00)
			535.30 (0.00)
			459.02 (0.00)
LR ($d = 0$)			
LR (one regime)	0.00 (1.00)	0.00 (1.00)	22.44 (0.00)
	15.80 (0.00))	19.34 (0.00)	576.10 (0.00)
	19.22 (0.00)	11.14 (0.00)	18.06 (0.00)

Notes: 1. The table presents likelihood ratio (LR) test statistics and their associated p -values in parentheses. 2. For each item reported in column 1, row one in column 2 and subsequent columns for that item presents statistics for CDR-Augmented, row two for CDR-Switching, and row three for CDR-Switching models. 3. LR (one regime) is a test for linear conditional mean dynamics. The null hypothesis is

$\mu_1 = \mu_2$, $\gamma = 1$, and the corresponding autoregressive coefficients in the two regimes are equal. 4. See notes 3-5 in Table 4.1.

Based on the different versions of the two classes of switching models used, we tested linearity using homoskedastik Gaussian models, homoskedastik stable models, and GARCH stable models. As per the null hypothesis of linearity in conditional mean, the unconditional means in the two regimes μ_1 and μ_2 are equal, the scale ratio γ is equal to one, and the corresponding autoregressive coefficients in the two regimes, if present in the specific parameterizations of the model versions used in the test, are equal. Accepting the null hypothesis, we end up with one regime only. Alternatively, we have two distinct regimes describing the stock returns growth rates.

4.2. Neural network test

Neural network test compares forecasts from linear model to forecasts from a neural network model. A test statistics constructed from the residuals of these models is basis for significance of the test results. A hypothesis for this test is linearity versus an alternative hypothesis of nonlinearity.

4.3. Empirical results on hypothesis tests

The empirical results for hypothesis tests pertaining to neural networks are presented in Table 5. Other hypotheses tests for augmented and switching models listed above are reported in Table 4 respectively, for the CDR-Augmented, CDR-Switching and SETAR Switching models. All tests are based on the likelihood ratio (LR) test statistic. In Table 4, a different test is reported in the various rows of the first column. For each such test, the numbers in the three rows in the other columns for that test are the LR test statistics for the three models estimated i.e. CDR-Augmented, CDR-Switching, and SETAR-Switching models. *P*-values are reported in parentheses. All statistical inferences are drawn at the five percent significance level.

4.3.1. Results on normality. The test for normality is easily rejected for data series under study. The test results do not change when we account for conditional heteroskedasticity. The statistical inferences

remain unchanged when we switch from 5 to 10 percent significance level.

4.3.2. Results on homoskedasticity. From Table 4 it transpires that when we test for homoskedasticity using null hypothesis homoskedasticity is rejected. The statistical inferences remain unchanged when we switch from 5 to 10 percent significance level.

4.3.3. Results on linearity in conditional mean. Our results show a strong evidence of nonlinearities in the conditional mean dynamics of U.S. stocks returns. Accounting for conditional heteroskedasticity seems unimportant when testing for linearity. Statistical inferences are not affected much when switching from 5 to 10 percent significance level.

4.3.4. Results from neural network tests. Results from neural network nonlinearity tests constructed using in-sample forecasts are presented in Table 5.

Table 5: Neural Network Tests Results

Data Series	Full Sample Excess Returns	Post War II Excess Returns	Post War II Real GDP
In-sample test statistics	2.615	1344.667	508.201
p-values	(0.000)	(0.000)	(0.000)
Out-sample test statistics	650.087	2907.700	270.897
p-values	(0.000)	(0.000)	(0.000)
In-sample RMSE	0.532	0.109	0.117
Out-sample RMSE	0.229	0.096	0.155

Notes on Table 5. 1. Column 2 show test statistics based on in-sample as well jackknife out-of-sample excess returns, column 3, shows such statistics for post World War II sample on excess returns and column 4 shows test statistics based on post World War II sample on real GDP. 2. In this Table for example row 1 column 2 shows neural networks nonlinearity test based on in sample forecast from a neural network versus a linear model. 3. P-values for each test statistics are shown below each test statistics in parenthesis. 4. The Table also shows RMSE based on in-sample as well as jackknife out-of-sample forecast performance of neural network models employed in this study for all the sample periods. 5. Row 5, in column 2 shows in-sample RMSE for full sample excess returns and in the same column, row

6 shows RMSE from jackknife out-of-sample forecast for neural network models. Similarly, we show RMSE for other sample periods and series used in this study.

In this Table the first row in the second column shows test statistics constructed from in-sample forecasts for full sample stock returns and the second row shows test statistics from jackknife out-of sample forecasts for the same period. Similarly, rows one and two in the third column show test statistics respectively for in-sample and jackknife out-of-sample forecasts for later sample (Post World War II). Column four rows one and two show test statistics for in-sample and jackknife out-of-sample forecasts respectively for real GDP growth rates. P-values for each test statistics are shown in parentheses beneath each test statistics. The Table also shows root mean square error (RMSE) for each in-sample as well jackknife out-of-sample forecasts. For example row three column two shows in-sample RMSE for full sample period and row four in the same column shows RMSE calculated from jackknife out-of-sample forecasts for the same sample period.

Neural network tests based on in sample forecast show that nonlinearities do exist in full sample and post World War II U.S. stock returns as well as real GDP series. Similarly, results based on jackknife out-of-sample forecasts from neural networks also show that nonlinearities do exist in all the series studied. The Table also shows root mean squared errors (RMSE) based on in-sample as well as jackknife out-of-sample forecast. From these results it transpires that jackknife out-of-sample forecast approximated from neural networks is superior to in-sample forecasts. However, the opposite is true for U.S. real GDP growth rates. A plausible reason for that could be a high stock return volatility during recessions and low stock return volatility during the expansion phase of business cycles.

4.3.5. Nature of stock returns. Our results based on in-sample as well as out-of-sample forecasts from neural networks models show a statistically significant evidence of nonlinearities in full sample U.S. stock returns. Similarly, our neural network results (in-sample as well as out-sample) based on later sample (post World War II era) period shows a statistically significant evidence of nonlinearities in

U.S. stock returns and real GDP growth rates. However, the magnitude of nonlinearities in U.S. stock returns post in World War II sample is substantially higher than the magnitude of nonlinearities in U.S. real GDP growth rates. The results from time series switching models show that the stock return volatility in lower regimes (recessions) is higher than in the higher regimes (expansions). These findings reveal presence of low stock return volatility during the expansions and high stock volatility in recession phase of the business cycles. In addition, the results from CDR-Augmented models show that the negative shocks are less persistent than the positive shocks which shows that expansions are of longer durations than the contractions. Generally our results show that variability in stock returns is greater than real GDP growth rates, however, stock returns and real GDP growth rates are pro-cyclical coincident variables that are positively correlated based on later sample stock returns and real GDP growth rates. These results are in line with previous studies including Beaudry and Koop (1993) McQueen and Thorely (1993), Brannas and De-Gooijer, (1994), and Peraz-Quiros and Timmermann (2001).